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ORIGINAL RESEARCH PAPER

Distribution network hosting capacity assessment: Incorporating probabilistic harmonic distortion limits using chance constrained optimal power flow

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Abstract

The capacity of renewable distributed generation (DG) connected in distribution networks is increasing. Use of power electronic interfaces means DG can inject harmonic currents through the point of common coupling into upstream networks. The limits stipulated in harmonic emission standards may create challenges for accommodating DG. To explore the impact of harmonic regulations on the ability of distribution networks to host DG, this work incorporates harmonic voltage constraints into a network hosting capacity assessment. A novel hosting capacity assessment approach is presented, incorporating percentile-based harmonic compliance levels as chance constraints over multiple periods into AC optimal power flow. The case study shows that network hosting capacity for DG could be evidently lower under rigorous compliance with harmonic distortion limits, but that relaxation of the risk constraints has significant value. Furthermore, the complex inter-connectivity between DG sites means that voltage, thermal and harmonic constraints all influence the locational feasibility for DG capacity.

KEYWORDS

distributed power generation, hosting capacity analysis, optimisation, power generation planning, power quality, power system harmonics, probability

1 | INTRODUCTION

Connecting renewable distributed generation (DG) to distribution networks requires compliance with a range of technical requirements, which are assessed by distribution network operators (DNOs) when a developer makes a connection request. Ensuring operation within statutory requirements and other applicable limits constrains the ability of the network to 'host' DG. While this issue is relatively well understood for voltage and power flow limits, this is not the case for harmonic current emissions from the power electronic converters of renewable DG identified in a range of harmonic studies [1–3]. The non-sinusoidal current injected from DG may increase voltage distortion in the network to inappropriate values.

Traditionally, harmonic studies in the distribution network have focussed on the measurement and management of harmonic voltage distortion from individual non-linear loads [4–8]. Well-accepted component models, simulation methods and analysis procedures have been developed. These include harmonic frequency scan [9], harmonic power flow simulation [10] for propagation studies and the design and placement of harmonic filters for mitigation. These approaches are generally based on networks without DG, and the load configurations are known.

More recently, harmonic emission assessment and mitigation techniques for renewable DG have been active research fields: [11] presented a PV system model to study harmonic coupling with the grid; [12] developed a wind turbine harmonic model for transient study; [13] used linearised models of converter control loops to analyse harmonic stability and internal resonance in wind farms; [14] used a sensitivity approach to assess wind farm harmonics [15] and studied aggregation and amplification of harmonics within a wind farm. All focus was on harmonic evaluation during the operation of specific generators.

Standards stipulate compliance limits for harmonic emissions (e.g. EN 50,160 [16]) and approaches to assess them, for example IEC-61,000-3-6 [17]. The UK standard, Engineering Recommendation (ER) G5/5 [18], is more aligned with [17] than its predecessor G5/4 [19]. For DG connection applications, compliance is evaluated in terms of total harmonic distortion voltage (THD) and maximum distortion for individual voltage harmonic orders (IHD). These are assessed at the point of common coupling and surrounding buses, especially when high background harmonic distortion is already present. Harmonic analysis is conducted separately from the analysis of voltage and thermal limits. In the UK unbundled market, connections are considered on a first come, first served basis; where these would result in harmonics above recommended limits—i.e. exceeding the harmonic 'headroom' between the background distortion and limits—DNOs will mandate the connection of filters, with the expense borne by the DG developer.

The rapid development of DG makes the evaluation of the network's capacity an important and recurrent problem. It is important to study harmonics from the perspective of DG connection evaluation in order to ensure the best use of network capacity. In this context, it is beneficial to consider harmonic studies at the initial stage of connection studies instead of being performed at a secondary stage. Clear guidance is needed to determine where network capacity exists to accommodate DG that is compliant with harmonic distortion limits as well as other technical considerations. The concept of network *hosting capacity* provides a useful framework for this. Hosting capacity is the maximum capacity of one or more DGs that may be connected in a network under specific constraints. These include a range of technical constraints such as voltage rise, thermal limits [20], voltage step, fault level [21] and security [22]. While not a direct replacement for long term planning, nor able to internalise fundamental uncertainties, hosting capacity provides a valuable framework for understanding how DG characteristics and other factors (e.g. controls) influence the effective use of the network. A range of techniques has been applied in formulating the hosting capacity problem, with the physical limitations of the network generally formulated as constraints.

There is a relatively small amount of work considering harmonic effects on hosting capacity, with [23] highlighting the issue. In [24], a circuit theory-based method is employed to estimate the capacity of DG that can be connected at a selected bus without exceeding acceptable harmonic distortion levels. It built an equivalent harmonic nodal model of the system at a connection point rather than the wider network study, therefore, limited to nodal analysis with the 'worst' and 'best' cases corresponding to the degree of harmonic current summation. A similar analytic approach is also used in [25–27]. [28] presented a data-driven method to reduce the need for complex harmonic modelling involved in the analytic approach, but its application in the planning stage could be limited due to the requirement of a large amount of DG measurement data. Nevertheless, [24] provides a valuable discussion of the main challenges in defining hosting capacity with harmonic analysis, which include as follows: the effects of variation in DG output on emissions, the need to apply the probabilistic compliance levels and the importance of appropriate time-averaging windows. A great portion of relevant work has used evolutionary algorithms with harmonic power flow calculations as subroutines to search the hosting capacity, for example, genetic algorithm (GA) [29–32], particle swarm optimisation (PSO) [33–35], multi-objective grasshopper optimisation algorithm (MOGOA) [36] and biogeography-based optimisation (BBO) [37]. A non-linear optimisation model for PV harmonic evaluation is used in [38] in conjunction with the Monte Carlo simulation (MCS) to capture time-varying load and PV output. However, with the exception of the recent study in [33], none of them has explicitly considered probabilistic harmonic compliance levels. While [33] successfully demonstrated the importance of probabilistic harmonic evaluation in a simple 3-bus test system, it did not consider time-varying renewable output and load, and the adopted Monte Carlo and hybrid particle swarm optimization and gravitational search algorithm approach could face scalability challenges in actual networks due to the large number of iterations required. Clearly, more research is needed to address the challenges in this topic. The authors' previous work [39] incorporates harmonic voltage constraints into a single-period harmonic optimal power flow (OPF) to determine the impact of harmonic regulation on hosting capacity but only considers the worst case 'firm' harmonic limits for IHD and THD in a single snapshot analysis.

To highlight the features and gaps in existing research, Table 1 offers a summary of the recent literature listed above in terms of key aspects of addressing a harmonic issue in hosting capacity study (The appendix offers a more detailed analysis). What is lacking in the previous work is a general hosting capacity framework able to handle the subtleties of harmonic analysis within a wider network setting. It is important that harmonic and non-harmonic aspects are studied together, rather than separately, as it is not known a priori whether the 'worst case' conditions from the perspective of voltage profile or power flow are the same as those which apply to harmonic emissions. Harmonic emissions from DG are variable, as are the underlying background emissions from the connected load, resulting in complex interactions. As such, it is important to account for variations over longer periods (ruling out 'snapshot' approaches), which aligns with the needs of fundamental network analysis where variations in DG output and demand govern the binding voltage and thermal limit constraints. Handling time variation is also important in aligning the analysis with probabilistic limits suggested by G5/5 and IEC-61,000-3-6. Finally, the need to make the best use of overall network capacity means that the framework must be able to handle *multiple* locations such that the potential trade-offs can be clearly understood and communicated.

Given these needs, this study presents a generalised hosting capacity evaluation framework with a focus on the impact of DG harmonics on the effective use of network by directly integrating probabilistic harmonic limits with time-varying patterns of DG and demand. As shown in the comparison table, the main contributions of the present study are as follows:

- Introducing a comprehensive optimisation framework for evaluating DG hosting capacities, which simultaneously

TABLE 1 Comparison of proposed and previous studies

Reference	[24]	[25]	[27]	[28]	[29]	[30]	[32]	[33]	[36]	[38]	[39]	This work
Modelling approach	ANP[1]	ANP	ANP	ANP	GA	GA	GA	MC and PSO GSA	MOGOA	NLP	NLP	AC-OPF and MINLP
Full load and renewable variation (As opposed to snapshot analysis for a single period)	✗[2]	✓	✗[2]	✓	✗[2]	✗	✓	✗	✓	✓	✗	✓
Evaluation of extensive constraints for HC[3](As opposed to only considering harmonic limits)	✗	✓	✗	✗	✓	✗	✗	✓	✗	✗	✗	✓
Full network study (As opposed to nodal analysis limited at a single location)	✗	✗	✗	✓	✗	✓	✓	✗	✗	✓	✓	✓
Interaction between multiple DGs (as opposed to single DG)	✗	✗	✗	✗	✗	✓	✓	✗	✗	✓	✓	✓
Harmonic probability	✗	✗	✗	✗	✗	✗	✗	✓	✗[4]	✗	✗	✓

Abbreviations: ACOPF, alternating current optimal power flow; GA, genetic algorithm; MC, Monte Carlo simulation; MINLP, mixed integer nonlinear programming; MOGOA, multi-objective grasshopper optimisation algorithm; PSOGSA, hybrid particle swarm optimization and gravitational search algorithm.

[1]ANP stands for Analytic and Non-optimal Approach. NLP is nonlinear optimisation.

[2]Snapshot analysis but extend to including best and worst possible condition (in-phase or anti-phase).

[3]HC stands for hosting capacity. Extensive evaluation means to study more than just voltage and its distortion.

[4]"Probabilistic" in the work refers to demand and renewable variation but not explicitly extending to harmonic probability in percentile levels.

evaluates network constraints at fundamental and harmonic frequencies covering long periods, as opposed to existing works where harmonic power flow simulation was mainly used in snapshot manner and not incorporated into the optimisation problem;

- The explicit linking of hosting capacity with harmonic emissions from multiple renewable DG in the network study, taking into account interactions with background harmonics from the non-linear load and renewable DG;
- The implementation of probabilistic harmonic compliance levels as non-firm limits using novel chance constrained optimisation methodology, reflecting IEC standards that allow short periods of harmonic constraint relaxation within a long evaluation period.

This study is structured as follows: Section 2 presents the optimisation formulation for the hosting capacity problem with harmonic limits. A case study of a medium voltage distribution network is analysed in Section 3, followed by a discussion and conclusion.

2 | HOSTING CAPACITY WITH HARMONIC LIMITS

Within the unbundled distribution business in Europe, DNOs cannot own DG and their remit only considers power delivery in terms of ensuring that the network can physically handle power flows with acceptable reliability, quality and safety. As such, hosting capacity offers valuable information on the best use of network capacity. Non-linear AC OPF techniques have been widely used to find network hosting capacity subject to a range of technical limitations. In this study, this is extended to determine harmonic-constrained hosting capacity accounting for the variability and coincidence of demand and generation levels. Hosting capacity can be viewed as an optimisation where the objective is to maximise the overall capacity of generation that can be connected at multiple locations:

$$\max \sum_{g \in G} p_g \tag{1}$$

where the decision variable p_g is the rated power capacity (MW) of each DG g determined across a set of time periods M (indexed by m, duration of each period is τ_m). The optimisation is subject to two sets of multi-period constraints: (1) network limits at fundamental frequency and (2) *additional* harmonic distortion limits. The network constraints that apply to the fundamental frequency are presented first followed by the more significant aspects of modelling harmonic constraints.

2.1 | Network constraints at fundamental frequency

The basic constraints represent the standard network physical limits at the fundamental frequency that apply in all periods m. These include voltage limits at each bus b (B, set of buses)constrained by maximum/minimum levels $V_b^{(+,-)}$:

$$V_b^- \le V_{b,m} \le V_b^+ \qquad \forall b \in B \tag{2}$$

Thermal loading limits on the flow at each end of lines and transformers, l (L, set of branches) are given by:

$$\left(f_{l,m}^{(1,2),P}\right)^2 + \left(f_{l,m}^{(1,2),Q}\right)^2 \le \left(f_l^+\right)^2 \qquad \forall l \in L \tag{3}$$

where f_l^+ is the branch apparent power flow limit; $f_{l,m}^{(1,2),P}$ and $f_{l,m}^{(1,2),Q}$ are the active and reactive power injections at each end (1,2) of the branch, governed by Kirchhoff's voltage law:

$$f_{l,m}^{(1,2),(P,Q)} = f_{l,(1,2),m}^{KVL(P,Q)}(\mathbf{V}_m, \delta_m) \quad \forall l \in L \tag{4}$$

where $f_{l,(1,2),m}^{KVLP}(\mathbf{V}_m, \delta_m)$ and $f_{l,(1,2)}^{KVLQ}(\mathbf{V}_m, \delta_m)$ are standard Kirchhoff's voltage law expressions, detailed as follows, with V_m and δ_m, respectively, depicting voltage magnitude and angle.

$$\begin{gathered} f_{l,m}^{1,P} = g_l \cdot V_{\beta_l^1,m}^2 - V_{\beta_l^1,m} \cdot V_{\beta_l^2,m} \cdot \\ \left[g_l \cdot \cos\left(\delta_{\beta_l^1,m} - \delta_{\beta_l^2,m}\right) + b_l \cdot \sin\left(\delta_{\beta_l^1,m} - \delta_{\beta_l^2,m}\right) \right] \end{gathered} \tag{5}$$

$$\begin{gathered} f_{l,m}^{1,Q} = -b_l \cdot V_{\beta_l^1,m}^2 - V_{\beta_l^1,m} \cdot V_{\beta_l^2,m} \cdot \\ \left[g_l \cdot \sin\left(\delta_{\beta_l^1,m} - \delta_{\beta_l^2,m}\right) - b_l \cdot \cos\left(\delta_{\beta_l^1,m} - \delta_{\beta_l^2,m}\right) \right] \end{gathered} \tag{6}$$

$$\begin{gathered} f_{l,m}^{2,P} = g_l \cdot V_{\beta_l^2,m}^2 - V_{\beta_l^2,m} \cdot V_{\beta_l^1,m} \cdot \\ \left[g_l \cdot \cos\left(\delta_{\beta_l^2,m} - \delta_{\beta_l^1,m}\right) + b_l \cdot \sin\left(\delta_{\beta_l^2,m} - \delta_{\beta_l^1,m}\right) \right] \end{gathered} \tag{7}$$

$$\begin{gathered} f_{l,m}^{2,Q} = -b_l \cdot V_{\beta_l^2,m}^2 - V_{\beta_l^2,m} \cdot V_{\beta_l^1,m} \cdot \\ \left[g_l \cdot \sin\left(\delta_{\beta_l^2,m} - \delta_{\beta_l^1,m}\right) - b_l \cdot \cos\left(\delta_{\beta_l^2,m} - \delta_{\beta_l^1,m}\right) \right] \end{gathered} \tag{8}$$

Nodal active and reactive power balances are modelled as ($\forall b \in B$).

$$\sum_{l \in L} p_{b,m}^{\mathrm{L}} + d_b^P \eta_m = \sum_{g \in G_b} p_g \omega_m + \sum_{x \in X_b} p_{x,m} \tag{9}$$

$$\sum_{l \in L} q_{b,m}^{\mathrm{L}} + d_b^Q \eta_m = \sum_{g \in G_b} p_g \omega_m \tan\left(\phi_{g,m}\right) + \sum_{x \in X_b} q_{x,m} \tag{10}$$

where $(p,q)_{b,m}^{\mathrm{L}}$ are the total active and reactive power injections into lines at bus b; $d_b^{(P,Q)}$ are the peak demands at the same bus; and $(p,q)_{x,m}$ are the total power supplied from upstream connections, that is the Grid Supply Point (GSP) substation. G_b and X_b are, respectively, the set of generators and upstream supply connected to bus b; ω_m is the generation level relative to its full capacity determined by the variable renewable resources in the period; η_m is the demand level relative to the overall peak value, and $\phi_{g,m}$ is the DG power factor angle. For nodes where DG is not to be connected, the decision variable p_g is fixed as zero.

The GSP substation is taken as the reference bus b_0 with voltage angle $\delta_{b_0,m} = 0$. The import/export limit for external connections x (for a set of GSP or internal sources) are modelled as:

$$\left.\begin{aligned} p_x^- \le\ & p_{x,m} \le p_x^+ \\ q_x^- \le\ & q_{x,m} \le q_x^+ \end{aligned}\right\} \quad \forall x \in X \tag{11}$$

2.2 | Firm and probabilistic harmonic constraint

Harmonic constraints are considered using voltage-related distortion limits to describe the allowed deviation from the nominal sine wave voltage form. Within the optimisation, the harmonic power flow analysis is performed to find the non-fundamental harmonic voltages of the system. Harmonic constraints can be formulated using either a 'firm' or probabilistic approach. The expansion of the variables and constraints at fundamental frequency to a range of selected harmonic frequency orders is achieved by combining the multi-period fundamental-frequency model as indexed by period m, with an additional set of harmonic variables and constraints, indexed by h (H, set of harmonic orders). To distinguish harmonic frequency f_h from fundamental frequency f_{base}, only harmonic variables and parameters are denoted by harmonic order $h = f_h/f_{base}$.

2.2.1 | Firm harmonic distortion constraints

Firm constraints require that harmonic levels cannot exceed agreed limits at any point in time. To ensure that the maximum DG capacity is compliant with harmonic regulations, both IHD and THD are formulated as hard constraints on harmonic voltage levels:

$$\frac{V_{b,m,h}}{V_{b,m}} \le IHD_h^+ \qquad \forall b \in B,\ h \in H,\ m \in M \tag{12}$$

$$\frac{\sqrt{\sum_{h=2}^{H} \left(V_{b,m,h}\right)^2}}{V_{b,m}} \le THD^+ \qquad \forall b \in B,\ m \in M \tag{13}$$

where $V_{b,m,h}$ are the harmonic voltages and $V_{b,m}$ is the bus voltage at fundamental frequency; IHD_h^+ and THD^+ are the planning level of harmonic distortion limits in the standards.

$V_{b,m,h}$ is obtained by performing a harmonic power flow analysis. In the matrix nation, it is solved as.

$$\left[V_{m,h}\right] = \left[Z_{m,h}\right]\left[I_{m,h}\right] \tag{14}$$

where for each harmonic order h in the period m, $[Z_{m,h}]$ is the network harmonic impedance matrix; $[I_{m,h}]$ is the vector of nodal harmonic current injection; and $[V_{m,h}]$ is the corresponding harmonic voltage vector including values for each bus. This enables the abnormally high voltage distortion associated with harmonic resonance to be captured, should the natural frequency of the network correspond to the frequency of a source of harmonics, with high elemental values in $[Z_{m,h}]$ and $[I_{m,h}]$ appearing at the same harmonic order h.

For buses with a DG connection, the harmonic current injection $[I_{b,m,h}]$ is dependent on DG emissions $I_{g,m,h}$ and is ultimately determined by DG capacity. Thereby, the optimisation variable p_g is optimised for constraints of fundamental and harmonic frequencies of interest. $I_{b,m,h}$ is described in Section 2.3.

By adding IHD and THD as firm constraints, hosting capacity is optimised strictly within the harmonic limit at all time periods, which provides completely risk-averse results. On the other hand, this can be considered conservative from a practical standpoint, given that harmonic violations could be less problematic than voltage and thermal violations. Therefore, it is valuable to consider probabilistic compliance levels.

2.2.2 | Probabilistic harmonic constraint

For an evaluation study at the planning stage, the IEC and UK regulations [17, 18] employ probabilistic compliance levels defined as 95% of all time periods. As [18] is not a statutory regulation, compliance with limits is ensured through an agreement between the DNO and DG developer, and there may be situations where the firm planning levels can be relaxed.

The required probabilistic level of compliance is modelled using chance constraints, with harmonic distortion allowed to violate limits for short durations over the whole study period but not beyond a defined percentile level. This is achieved by restating firm harmonic distortion constraints (12) and (13) as (15) and (16), together with the additional (17):

$$\psi_m \frac{V_{b,m,h}}{V_{b,m}} \le IHD_h^+ \qquad \forall b \in B,\ \ h \in H,\ \ m \in M \tag{15}$$

$$\psi_m \frac{\sqrt{\sum_{h=2}^{H} \left(V_{b,m,h}\right)^2}}{V_{b,m}} \le THD^+ \qquad \forall b \in B,\ \ m \in M \tag{16}$$

$$\sum_{m \in M} \psi_m \tau_m \ge \varepsilon \sum_{m \in M} \tau_m \tag{17}$$

where ψ_m is a binary variable indexed by period m; when its value is set to zero, means that the certain period is excluded from THD and IHD calculation in Equations (15) and (16). In other words, harmonic distortions in that period (when $\psi_m = 0$) are relaxed from strict harmonic compliance. Only when ψ_m is with a non-zero value of 1, the harmonic limits in Equations (15) and (16) are enforced at the corresponding periods. While a small group of periods are allowed to exclude from the strict harmonic limits, the occurrence of such events over the whole period should be limited to a certain percentage, as suggested in the regulation. This is enforced by constraint (17), where ε is the minimum percentile (e.g. 95%) of harmonic-compliant periods (with $\psi_m = 1$) against the total periods (as $\sum_{m \in M} \tau_m$). By optimising the binary value of ψ_m, periods with short durations but high harmonic distortions can be deliberately excluded from the harmonic compliance assessment, allowing for a further increase in hosting capacity. If required, a reasonable limit could also be applied to provide a maximum allowable relaxed value of IHD_h^{relax} and THD_h^{relax}, so the periods that are excluded from Equations (15) and (16) do not experience unlimited harmonic distortion levels.

2.3 | Modelling harmonic network and harmonic sources

In Equation (14), the evaluation of bus harmonic voltage is derived from the superposition principle of electrical circuit theory for each harmonic order. The harmonic impedance matrix of the network $[Z_{m,h}]$ is determined by network circuit parameters and topology data. Due to the frequency-dependent network components (such as inductance and capacitance of lines) and the varying loading state of harmonic sources that change the equivalent impedances and harmonic current emission, $Z_{m,h}$ and $I_{m,h}$ are indexed by both harmonic order h and period m. Figure 1 shows $Z_{m,h}$ in its *multi-period* and *multi-frequency* form. By choosing appropriate harmonic models of network components, $[Z_{m,h}]$ and $[I_{b,m,h}]$ can be derived using network parameters prior to the optimisation.

Different approaches have been proposed to model linear and non-linear components at harmonic frequencies [7, 10, 40]. These vary in complexity and data requirements. It is appropriate to note that at the DG planning stage, simplified models are acceptable, given that data, such as the harmonic emission spectrum, internal design and operating conditions may not be fully known. Moreover, given that this work is *explicitly not* about advancing the state-of-the-art of detailed harmonic modelling of particular equipment or devices, it is reckoned that the simplified component models can help to clarify and generalise the whole optimisation framework. The adopted harmonic model of network components, including feeder, transformer, passive and non-linear load and DG is presented as follows.

Network lines are represented by a single phase pi-circuit with constant resistance $R_{l,h} = R_l$ and frequency-dependent inductance $X_{l,h}^L = X_l^L$ and capacitance $X_{l,h}^C = X^C/h$. R_l and X_l are the values for line l at the fundamental frequency. Long-line effects and skin effects are generally neglected in short distribution feeders.

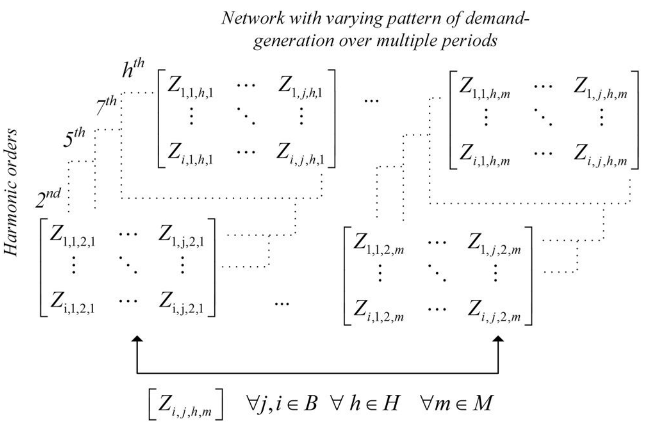

FIGURE 1 Formulation of network harmonic impedance matrix

The main characteristics of a transformer that affects harmonic flows are the short-circuit impedance and winding connection type. It is acceptable in a planning study to model transformers (indexed by tmr) as series-connected impedance with constant $R_{tmr,h} = R_{tmr}$ and frequency-dependent inductance $X_{tmr,h} = X_{tmr}h$ terms.

Aggregated load values (MW and Mvar) at medium voltage level buses are usually readily available. For the passive load at bus b (peak value as d_b^{linear} and $d_b = d_b^{linear} + d_b^{nonlinear}$), a simplified parallel form representation is adopted in this work [41]:

$$R_{b,h}^{load} = \frac{V_{b,m}^2}{d_b^{P,linear}\eta_m}$$
$$X_{b,h}^{load} = \frac{V_{b,m}^2}{d_b^{Q,linear}\eta_m} \cdot h \tag{18}$$

where $R_{b,h}^{load}$ and $X_{b,h}^{load}$ is the equivalent harmonic impedance of linear load at the harmonic order h; $V_{b,m}$ is the fundamental voltage determined by AC power flow (Section 2.1); $d_b^{P,linear}$ and $d_b^{Q,linear}$ represent the active and reactive part of passive load at the fundamental frequency, respectively, and η_m is the demand level relative to the overall peak value at period m.

When the load also has a considerable non-linear part, it is reasonable in the context of DG planning to present this non-linear component of loads as harmonic current sources, which cause background distortions. The current source is connected with the passive linear part in parallel as shown in Figure 2. Its harmonic current value $I_{b,m,h}^{nonlinear}$ can be calculated using its harmonic emission spectrum $I_{b,h}^{spectrum}$ against its fundamental current as

$$I_{b,m,h}^{nonlinear} = I_{b,h}^{spectrum}\left(d_b^{nonliner}\eta_m \middle/ V_{b,m}\right) \tag{19}$$

where $d_b^{\mathrm{nonlinear}}$ is the peak value of the non-linear load.

Harmonic emissions from both load and DG are taken into account for constructing nodal current injection in harmonic current vector $[I_{m,h}]$. The harmonic emission $I_{b,m,h}$ at the bus b is the total from non-linear load $I_{b,m,h}^{nonlinear}$ and DG $I_{g,m,h}$ (if any are located there):

$$I_{b,m,h} = I_{b,m,h}^{nonlinear} + \sum_{g \in G_b} I_{g,m,h} \tag{20}$$

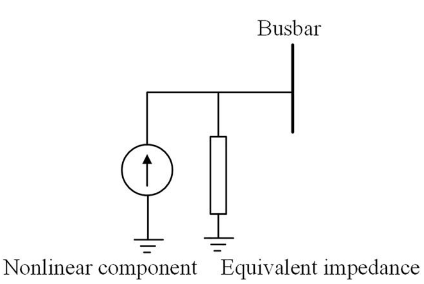

FIGURE 2 The parallel form model of linear and non-linear load

Harmonic current emission $I_{g,m,h}$ from DG g is formulated as:

$$I_{g,m,h} = I_{g,h}^{spectrum}\left(p_g \middle/ V_{g,m}\right)\omega_m \tag{21}$$

where $I_h^{spectrum}$ is the DG harmonic current spectrum based on its rated power, indicating the h th order *maximum* current emission for the DG technology and V_g is the fundamental nodal voltage at the DG bus. By relating these to the DG optimisation variable p_g and fundamental voltage $V_{b,m}$, DG harmonic emission is determined by the optimisation for each period m and harmonic order h. For illustrative purposes, $I_{g,m,h}$ is assumed to change linearly with DG output, but it is important to note that the actual harmonic current from DG could exhibit different characteristics (e.g. see [42]); therefore, the parameter of the harmonic current spectrum can be updated by taking into account its changes at different bands of power output rather than its rated power. It is also appropriate to note that at the DG planning stage, simplified models are acceptable given that data, like the dynamic harmonic emission spectrum, internal design and operating conditions may not be fully known or require significant effort to obtain.

2.4 | Overall structure, computational performance and implementation

Based on the proposed formulations, its *multi-period* (covering multiple time periods over a long time horizon to capture the variation of load and generation) and *multi-frequency* (covering both fundamental and harmonic frequencies) optimisation structure can be visualised in Figure 3. The highest value of h is set according to the harmonic regulations, often as 50 (i.e. up to 2500 Hz at 50 Hz base frequency). The highest value of m equals to the total number of representative periods that are generated using the time series aggregation process outlined in the following paragraph. The objective function, that is a single set of maximum DG capacities p_g, is constrained by both fundamental and harmonic limits in each period m. As the fundamental constraints modelled by AC power flow in Section 2.1 are non-linear and non-convex in nature, the whole formulation is a non-convex non-linear optimisation problem (NLP). Directly using half-hourly time series over a year will result in an NLP problem that is too large to be solved; since it would be a unique inter-period, inter-frequency set of generation capacity variables p_g being optimised covering 17,520 sets of non-linear constraints where each set includes constraints at all frequencies of interest.

An important feature of the framework in addressing the computational challenge of optimising a single object with non-linear and non-convex constraints over a great number of time steps, is the use of 'representative' demand and renewable resources level combinations as inputs, rather than half-hourly time series or 10-min series suggested by [16]. This involves DG output and demand time series being discretised into ranges as shown in Figure 4, which are then aggregated

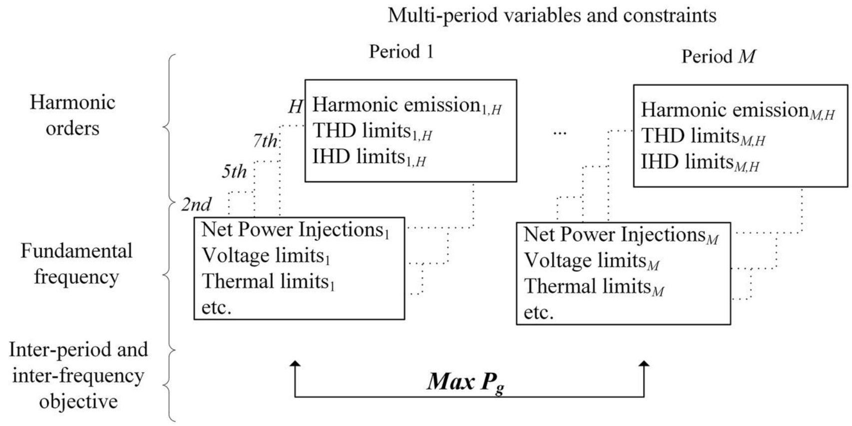

FIGURE 3 Structure of multi-period multi-frequency formulation

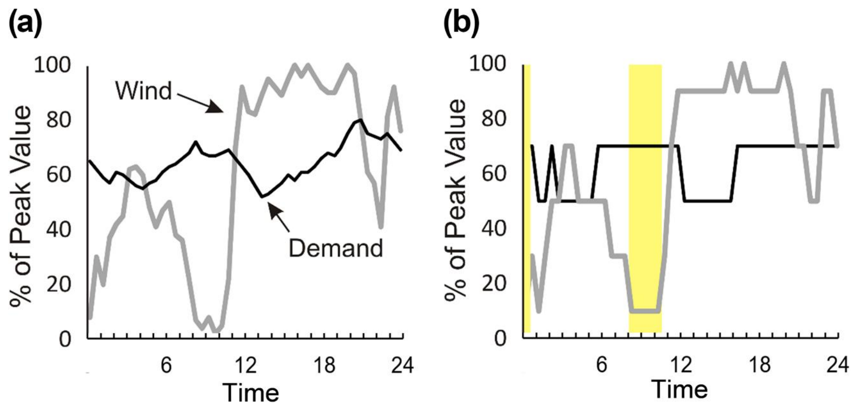

FIGURE 4 (a) Normalised hourly demand and wind power time series and (b) discretised wind and demand time series

according to their degree of coincidence. This effectively reduces the computational burden of this large non-linear programme and efficiently captures the probabilistic behaviour of DG and the full range of generation-demand combinations, including extreme cases (e.g. maximum generation-minimum demand). While discretisation has a minor impact on accuracy, it is small compared to uncertainties associated with other planning factors (e.g. cost, location and demand growth). A detailed treatment of discretisation and combination are given in [43].

When harmonic limits are defined probabilistically using the chance constraint approach, the optimisation formulation becomes a mixed-integer NLP. It is generally more computationally challenging but mature solvers that adopt well-known classic approaches (e.g. Outer Approximation [44]) exist for extremely large problems. Given the predominantly convex feature of the harmonic model adopted in this work, the main computational burden of the optimisation is not attributed to harmonic constraints when representing the limit as firm constraints. The treatment of discretising time series reduces the problem to a manageable scale.

The method is coded in the AIMMS optimisation modelling suite [45] on a PC (Intel i7, 2.1 GHz, 8 GB RAM). The firm harmonic constrained OPF uses the CONOPT 3.40 NLP solver, and the chance-constrained harmonic OPF uses the AIMMS Outer Approximation Algorithm (AOA) Solver, taking 5 and 30 min in the following case study, respectively. The computational time is acceptable for an off-line planning study.

3 | CASE STUDY

Figure 5 shows the one-line diagram of 16-bus 33-kV network from the UK Generic Distribution System [46] used as the case study. The maximum demand of the network is 38 MW with minimum levels 40% of that. The peak demand values at buses are shown in Figure 5. The feeders are supplied by two 30 MVA 132/33kV transformers. The GSP voltage is assumed to be nominal. The line parameters are provided in Table 2. Voltage limits are taken to be ±6% of nominal. A voltage regulator (VR) is located between buses 8 and 9, with a 1.03p.u. target voltage to ensure voltage compliance at the end of the feeder during high demand. The network has four potential locations (buses 7, 11, 12, 16) at which new wind farms (WFs) can be connected. There is no active network management considered, and all wind farms are operated at a constant unity power factor.

The hosting capacity is determined across a full year where half-hourly generation and demand data (as shown in Figure 6) were processed using the approach briefly mentioned in Section 2.4 to reduce the computational burden to 74 'representative' periods of generation and demand combinations. The non-harmonically constrained case is presented first,

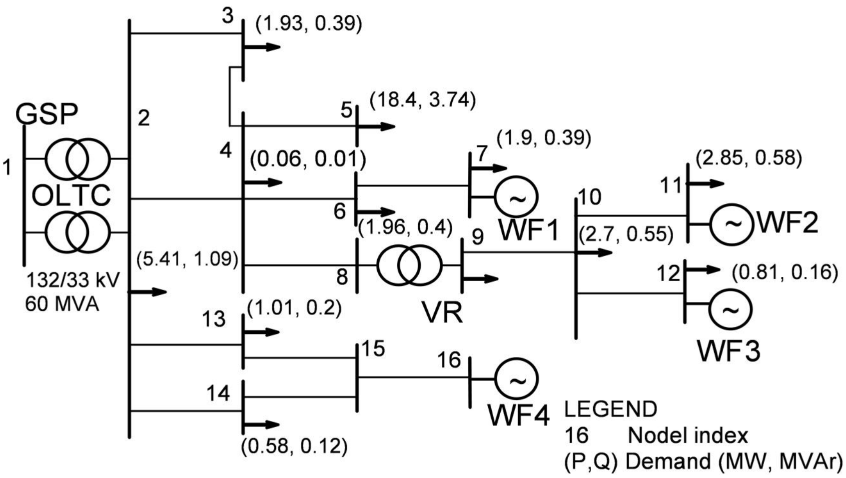

FIGURE 5 33 kV network one-line diagram with four wind farms

TABLE 2 Test network parameters: line parameters (system base for per-unit (pu) values are 100 MVA and 33 kV)

Start bus	End bus	R (pu.)	X (pu)
Bus_1	Bus_2	0.00	0.13
Bus_2	Bus_3	0.20	0.45
Bus_2	Bus_4	0.19	0.30
Bus_2	Bus_13	0.21	0.28
Bus_2	Bus_14	0.51	0.53
Bus_3	Bus_4	0.22	0.29
Bus_4	Bus_5	0.03	0.03
Bus_4	Bus_6	0.52	0.38
Bus_4	Bus_8	0.44	0.39
Bus_13	Bus_15	0.27	0.28
Bus_14	Bus_15	0.40	0.29
Bus_6	Bus_7	0.39	0.35
Bus_8	Bus_9	0.07	0.10
Bus_9	Bus_10	0.54	0.73
Bus_10	Bus_11	0.94	0.66
Bus_10	Bus_12	1.59	1.21
Bus_15	Bus_16	0.40	0.29

followed by a comparison with harmonic-constrained hosting capacity. The impact of relaxing firm harmonic constraints by different percentiles is also demonstrated. The harmonic voltage planning levels for 33kV systems in [18] are adopted as upper harmonic limits: 4% for voltage THD with IHD values shown in relevant figures.

3.1 | DG and harmonic sources

For the harmonic analysis, WFs are modelled as voltage-independent (non-coupled) current sources with the harmonic profile of a 2 MW wind turbine chosen for illustration (details are not provided due to commercial sensitivity). The spectrum of maximum harmonic current (measured at rated power) produced by this DG is presented in Figure 7. It extends up to the 50th order and shows relatively high distortion at lower (second–eighth) and higher harmonic orders (33rd–37th).

For background supply voltage distortion, the third, fifth and seventh harmonics are generally most severe in the distribution system. Here, the voltage distortions from non-linear load connected at buses around the WFs are given as constant background values: 1% at the third, fifth and seventh orders while others are considered small and neglected. These data can be updated where detailed measurement or modelling is available but are considered sufficient to illustrate the methodology. Harmonic currents from different sources are assumed to be in-phase resulting in the worst case with maximum distortion (although randomly assigned phases can be used).

3.2 | Non-harmonic hosting capacity

The network capacity to accommodate DG depends on its electrical characteristics and the pattern between variable renewable DG resources and local load. At low demand levels, a given DG output leads to more exported power and network voltage and thermal constraints that are more likely to be active, thus limiting hosting capacity.

The initial hosting capacity evaluation considers only voltage and thermal constraints and ignores harmonic limits, with the results presented in the second column of Table 3. It is evident that the network exports power to the GSP since the 43.4 MW total DG capacity surpasses maximum local demand by some margin. A substantial amount of capacity is available at buses 7 (WF1) and 16 (WF4) while lower amounts are allocated at buses 11 (WF2) and 12 (WF3). The constraints that actively limit the capacity at these locations are the upper voltage limits (1.06 p.u.) at WFs during the worst-case low demand period while the thermal limit is reached on line 15-16 (connecting WF4). The differences between WF1&4 and

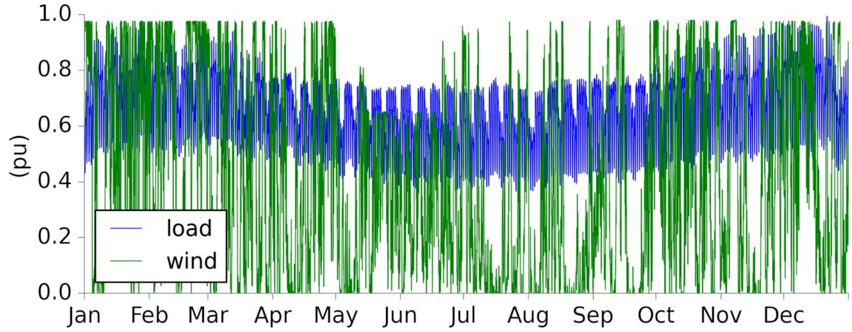

FIGURE 6 Half-hourly demand and wind variation

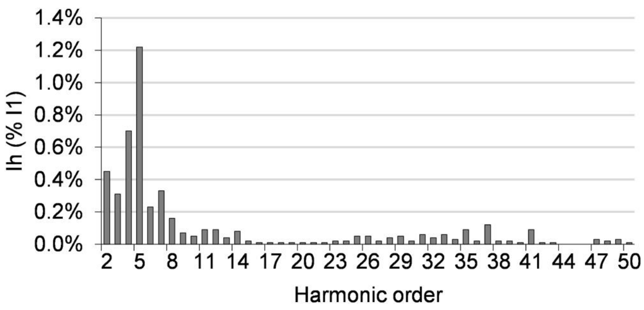

FIGURE 7 Maximum harmonic current produced by the wind turbine

TABLE 3 Hosting capacity with (HOPF) and without (OPF) harmonic constraints

Wind farm	OPF (MW)	HOPF (MW)	Reduction (%)
WF1	14.8	9.2	38%
WF2	9.2	4.9	47%
WF3	5.0	2.0	60%
WF4	14.4	10.6	26%
Total	43.4	26.7	38%

Abbreviations: HOPF, harmonic-constrained optimal power flow; OPF, optimal power flow.

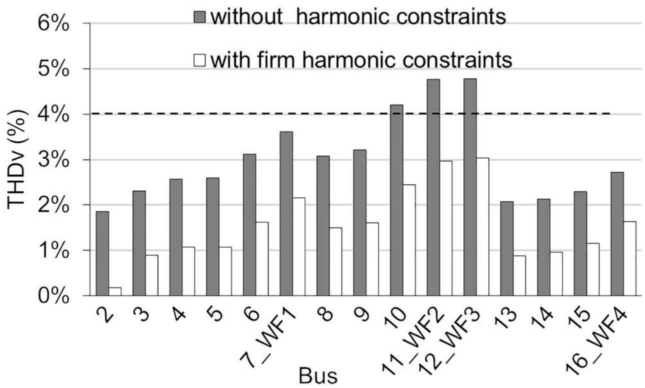

FIGURE 8 Total harmonic voltage distortion (THD) at each bus under different optimal DG capacity results

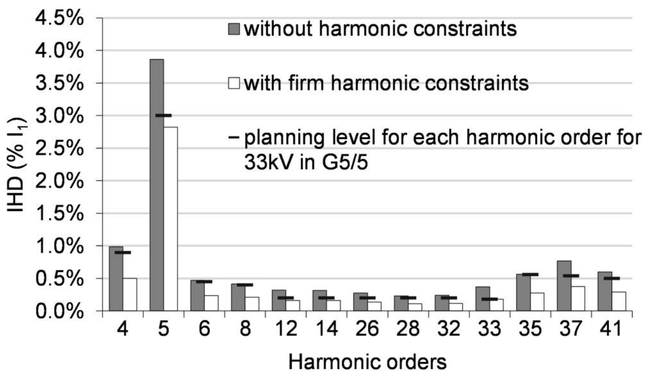

FIGURE 9 Harmonic voltage distortion for individual orders (IHD) at bus 12 under different optimal DG capacity results

WF2&3 are largely due to WF1&4 being closer to the GSP (i.e. lower line impedance and voltage sensitivity). The higher capacity of WF2 than WF3 is due to a shorter connection feeder and higher local demand.

3.3 | Compliance with harmonic limits

To examine whether the hosting capacity identified without considering harmonic limits is compliant with standards, harmonic power flow simulations were conducted with the 43.4 MW capacity and scaling injections by respective capacities. The results for THD at each bus are given as dark columns in Figure 8 with the limit given as a dotted line. THD at buses 11 (WF1), 12 (WF3) and 10 clearly violate the planning level, with the most severe violation at bus 12 (WF3). This bus also has high IHD at multiple individual harmonics, as shown in Figure 9, exceeding limits for 13 harmonic orders with the fifth, 33rd and 37th being the worst. Other less severe IHD violations occur at other buses. It is probable that the DNOs would seek harmonic filtering to be commissioned at WF3.

3.4 | Harmonic-constrained hosting capacity

With the initial hosting capacity failing to comply with the G5/5 harmonic limits, obtaining a standards-compliant planning capacity will be vital in understanding the influence of harmonics on DG capacity and the requirement for mitigation. Applying the harmonic-constrained OPF, a revised maximum

DG hosting capacity can be gained. The harmonic distortions are applied initially as firm limits so that distortions in each period must comply with the planning levels. The results in Table 3 show an overall DG capacity of 26.7 MW, a 38% reduction from the non-harmonically constrained case. This is a result of the harmonic constraints becoming active and limiting DG capacity to maintain harmonic compliance, rather than voltage and thermal limits alone. It is also notable that the changes are non-uniform with capacity at WF3 (bus 12) reduced by around 60%, while the capacity at WF4 (bus 16) reduced by only 26%.

Harmonic-constrained THD and IHD values are presented as white columns in Figures 8 and 9, respectively. The solution identified by the HOPF complies with harmonic limits, since both THD and IHD are directly embedded into the optimisation. Inspecting the result, the binding constraints that prevent the connection of further capacity are the 33rd order harmonics at all WF buses that reach the 0.18% distortion limit. This suggests that if harmonic filtering is selected as a mitigation measure, the 33rd order harmonic should be a priority.

3.5 | Probabilistic harmonic constraints

Since ER G5/5 (and IEC-6100-3-6) recommend probabilistic harmonic distortion compliance and as neither are statutory regulations, the firm harmonic constraints might be too onerous. The effect of adopting probabilistic compliance levels using the chance-constrained approach (Section 2.3) is examined by determining hosting capacity subject to a range of compliance percentiles from 100% (firm limits) to the 95% level of the IEC standards. Figure 10 shows that the harmonic hosting capacity generally increases with less strict compliance requirements with 6.6 MW more capacity available (an increase of 25%) at the 95% compliance level. However, the impact is complicated by the specific topology and resource-load patterns in this network. The increase of the hosting capacity is not linear with reducing compliance percentile. There is an immediate 2 MW of capacity released by relaxing the firm compliance period by one percentile to 99%, which implies that the period with the worst harmonic distortion lasts for a very short duration across the whole study period. The impact of relaxation also varies among the WFs, with WF2 benefiting most and WF4 the least, but these changes depend on the degree of relaxation. Although not shown in Figure 10, relaxation below 95% delivers hosting capacities that progressively resemble those for the non-harmonically constrained case. This complex capacity-releasing effect would be challenging to explore without an appropriate framework such as that presented here.

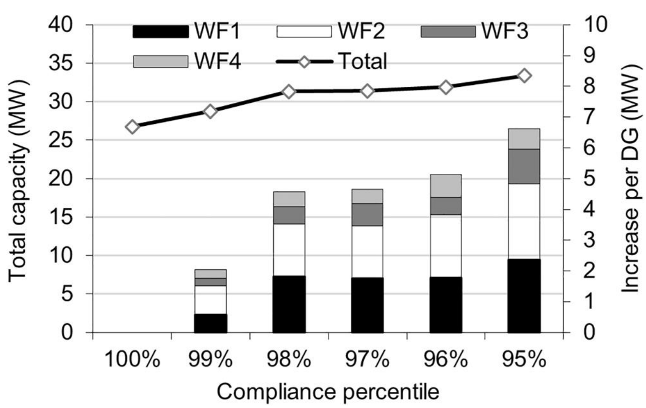

FIGURE 10 Hosting capacity with harmonic chance constraints under different compliance percentiles and the changes to each WF from 'firm' 100% case

4 | DISCUSSION

Output-dependent harmonic emission modelling of DG and load used in the assessment framework would facilitate the understanding of harmonically critical cases that are different from those at the fundamental frequency where low demand levels constrain the hosting capacity. For example, peak load may inject considerable harmonic current into the background distortion that actively constrains the harmonic hosting capacity. In practice, compared with constraint studies at the fundamental frequency, harmonic hosting capacity is more difficult to assess accurately due to data sufficiency and that the load model, the DG model and the aggregation of harmonics—all need certain assumptions. In the case study, DG harmonic emission is assumed to change linearly with its output based on the rated spectrum. DG emission could be more complex as variable output and impact of internal control schemes lead to dynamic and uncertain emissions characteristics. DG could also have interharmonic emissions (non-integer multiples of the fundamental frequency). To include interharmonic, the harmonic index h (as [2–50] in the formulation i.e. up to 2500 Hz based on 50 Hz) would be extended to be [2–500] that present the multiplier of 5 Hz. When such corresponding data are available, sophisticated models can be characterised and adopted. Nevertheless, one of the primary difficulties in applying sophisticated harmonic sources modelling is a lack of data and consequent uncertainty to parameterise it. The standards [17, 18] recognise this explicitly and suggest a pragmatic approach to represent the harmonic source using a selective linear scaling approach to approximate the effects of current summation/cancelation.

This study is primarily for relatively large DG at Medium Voltage (MV) network (1–35kV in the UK), since these MW-scale DG potentially have significant harmonic emission. MV networks are operated as a balanced system in the UK. Accordingly, we have selected a 33kV UK network to test the proposed methodology. While the unbalanced Low Voltage (LV) network is not within the main scope of this work, we think that the proposed method could still be applicable and useful with some modifications. When considering the collective harmonic impact of distributed small DG (such as rooftop PV) in LV network, the power flow part of the formulation we proposed in this study would need to be reformed to present the unbalance power flow. There are some recent developments in the topic of three-phase optimal power flow

[47, 48] that could be adopted in this light. We are interested to consider this in our future study.

MINLP (Mixed-Integer Non-linear) optimisation is used in this work because the full classic formulation of AC power flow at the fundamental frequency is non-convex and non-linear in nature. To effectively reduce the computational burden, the full time series of wind and demand availability is aggregated into a manageable number of representative wind/demand scenarios based on their joint probability of occurrence. We found the results are already satisfactory for an off-line planning study as the reduced-size MINLP can be solved in the acceptable time frame: 30 min for the typical UK network in the case study. While we did not implement any relaxation or linear approximation of the MINLP formulation to become convex, there is a recent study that has investigated the convexification of AC OPF, such as [49]. If it is necessary to further enhance the rapidity of the proposed method, this relevant reformation approach can be adopted.

Reduction in the network hosting capacity is indicated by the result in Table 3 after considering compliance with harmonic limits. More hosting capacity is released with probabilistic constraints considering percentile levels down to 95% as ER G5/5 suggests. In practice, the harmonic requirement is not as strongly enforced as voltage and thermal rating limits, and the worse-case scenarios among the whole study period may happen infrequently. As a result, there could be more 'space' for accommodating DG than reflected in connection agreements, but this will be case specific.

Harmonic filters can provide a mitigation solution to facilitate DG integration. The optimisation model from this study could be the basis for evaluating optimal filter placement and for cost-benefit analysis while ensuring a system-oriented view. The active constraints found in the HOPF analysis can indicate the order(s) at the buses that constrain DG. It can also be used to check the level of mitigation required, at which point the active constraint will switch from harmonics to other non-harmonic network constraints. Similarly, the evaluation of control techniques from active network management at the fundamental frequency (e.g. [20, 50]) can also be studied together with harmonic mitigation solutions using the proposed framework by formulating them as control variables.

5 | CONCLUSION

Connecting renewable DG is subject to limits on harmonic distortion. Assessing hosting capacity using standard approaches could fail to identify the violations of stipulated harmonic limits and potentially suggest impractical levels of DG capacity. This study presents a generalised optimisation framework, which enables renewable hosting capacity assessment by simultaneously integrating limits for both harmonic and fundamental frequency voltages and currents. In addition, a chance-constrained approach is introduced to quantify the additional network capacity that can be released by adopting probabilistic harmonic compliance levels, in line with international standards.

The case study shows that network hosting capacity for DG could be evidently lower under rigorous compliance with harmonic distortion limits, but that the relaxation of the risk constraints has significant value. Furthermore, the complex inter-connectivity between DG sites means that voltage, thermal and harmonic constraints all influence the locational feasibility for DG capacity. The explicit linking of hosting capacity with harmonic emissions from renewable DG provides valuable information to guide DG developers and DNOs in arranging harmonic-compliant connections and provides a basis for the identification of mitigation solutions.

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CONFLICT OF INTEREST

None.

PERMISSION TO REPRODUCE MATERIALS FROM OTHER SOURCES

None of copyrighted materials in this work.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the authors upon reasonable request.

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APPENDIX

TABLE A1 A detailed comparison of proposed and previous studies (extended version of Table 1 in the introduction section)

Ref	Modelling approach	Load and renewable variation	Full constraints evaluation for HC	Full networkor nodal analysis	Single or multiple DG locations	Probability
[24]	Equivalent single nodal analysis at PCC using circuit theory rather than full network analysis.	Snapshot analysis for best and worst possible condition (in-phase or anti-phase)	No, only consider harmonic	Norton equivalent harmonic model of the system at connection point	Single	No
[25]	Analytic approach	Full time-varying load and renewable	Voltage rise and harmonic distortion	Simplified network	Single DG location	Yes
[26]	Analytic approach	Snapshot of load and renewable	Only consider harmonic	Simplified 2-bus system at PCC	Single DG location	DG capacity uncertainty
[27]	Equivalent nodal analysis at PCC; calculation using the circuit theory	Three snapshot scenarios as the worst, best and moderate	No	Nodal analysis	Single	No
[28]	Data-driven approach that eliminate the need for complex harmonic modelling	Several days' data with time resolution in seconds	Not directly applicable for HC evaluation	Full network	None	No
[29]	Evolutionary algorithm (GA) for hosting capacity analysis with the subroutines of harmonic limit check	Two snapshot, best and worst possible condition	Also consider feeder overloading	Equivalent single point study of two bus system	Single	No
[30]	Genetic algorithm, (NSGA II) optimisation	Snapshot analysis	Only consider harmonic	Full network	Multiple	No
[32]	Genetic algorithm, optimisation	Representative demand scenarios with non-variable dg	Only voltage	Full network study	Multiple	No
[36]	Multi-objective grasshopper optimisation algorithm (MOGOA)	Typical days of load and renewable	Only consider harmonic	Equivalent single point study at PCC bus	Single location with multiple DG	Renewable uncertainty
[38]	Optimisation problem in conjunction with the Monte Carlo simulation (MCS)	Full time-varying load and renewable	Only consider harmonic	Full radial network	Multiple DG	Renewable uncertainty
This work	Embedded harmonic into optimal AC power flow as non-linear programing	Full half-hourly time series of both load and renewable over a year	Full evaluation of fundamental and harmonic constraints	Full network study using UK generic distribution network	Multiple locations	Harmonic limits as chance constraints

Note: Comprehensive evaluation means to study more than just voltage and its distortion. "Probabilistic" in the work refers to using time series to capture demand variation but not extend to harmonic probability.

Abbreviation: HC, stands for hosting capacity.